(8 pages)

Reg. No. : ....

Code No. : 5547 **Sub. Code : KMAM 42**

M.Sc. (CBCS) DEGREE EXAMINATION,
APRIL 2020.

Fourth Semester

Mathematics

COMPLEX ANALYSIS

(For those who joined in July 2016 only)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. Analytic functions are characterized by the condition

(a) $\frac{\partial f}{\partial x} = 0$ (b) $\frac{\partial f}{\partial z} = 0$

(c) $\frac{\partial f}{\partial \bar{z}} = 0$ (d) $\frac{\partial f}{\partial y} = 0$

2. The radius of convergence of the series $\sum \frac{z^n}{n!}$ is

(a) 1 (b) 0

(c) $\frac{1}{n}$ (d) ∞

3. A linear transformation carries circles into

(a) quadrilaterals (b) circles

(c) rectangles (d) straight lines

4. Consider a circle C with the centre a, represented by the equation $z = a + \rho e^{it}$, $0 \le t \le 2\pi$, then $\int_C \frac{dz}{z-a}$ is

(a) $2\pi i$ (b) 0

(c) 1 (d) 2π

5. The index of the point a w.r.t. the curve γ is

(a) $\int_\gamma \frac{dz}{z-a}$ (b) $\frac{1}{2\pi i} \int_\gamma \frac{dz}{z-a}$

(c) $\int_\gamma \frac{|dz|}{|z-a|}$ (d) $\frac{1}{2\pi i} \int_\gamma \frac{dz}{(z-a)^2}$

6. A function which is analytic and bounded in the whole plane must reduce to a constant. This theorem is known as

(a) Morera's theorem

(b) Fundamental theorem of algebra

(c) Liouville's theorem

(d) Cauchy's theorem

7. The residue of $\frac{e^z}{(z-5)(z-2)}$ at $z = 2$ is

(a) $\frac{e^2}{-3}$

(b) $\frac{e^2}{3}$

(c) $\frac{e^5}{3}$

(d) $\frac{e^5}{-3}$

8. If $\lim_{z \to a} f(z) = \infty$, the point '$a$' is said to be

(a) an isolated singularity

(b) a zero

(c) a pole

(d) an accumulation point

9. The residue of $\frac{z^2}{(z-1)(z-2)(z-3)}$ at $z=1$ is

(a) 1 (b) 2

(c) 0 (d) 1/2

10. If $z=e^{i\theta}$ then $\frac{1}{2i}\left(z-\frac{1}{z}\right)$ is

(a) $\cos\theta$ (b) $\sin\theta$

(c) $\cos 2\theta$ (d) $\sin 2\theta$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Verify Cauchy-Riemann's equations for the function z^3.

Or

(b) If all zeros of a polynomial $P(z)$ lie in a half plane, prove that all zeros of the derivative $P'(z)$ lie in the same half plane.

12. (a) Given three distinct points z_2, z_3, z_4 in the extended plane, prove that there exists a unique linear transformation S which carries them into 1, 0, ∞ in this order.

Or

(b) Complete $\int_{\gamma} x dz$ where γ is the directed line segment from 0 to $1 + i$.

13. (a) State and prove Morera's theorem.

Or

(b) State and prove Liouvelle's theorem.

14. (a) Define the following :

(i) isolated singularity of a function

(ii) zero of order h of a function

(iii) pole of a function

(iv) meromorphic functions

Or

(b) State and prove the maximum principle for analytic functioning.

15. (a) State and prove Rouche's theorem.

Or

(b) If u_1 and u_2 are harmonic in a region Ω, prove that

$$\int_{\gamma} u_1 * du_2 - u_2 * du_1 = 0$$ for every cycle γ which is homologous to zero in Ω.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If $u(x, y)$ and $v(x, y)$ have continuous first order partial derivatives which satisfy the Cauchy-Riemann differential equations, prove that $f(z) = u(z) + iv(z)$ is analytic with continuous derivative $f'(z)$ and conversely.

Or

(b) State and prove Abel's limit theorem.

17. (a) If $T_1z = \dfrac{z+2}{z+3}$, $T_2z = \dfrac{z}{z+1}$, find T_1T_2z, T_2T_1z and $T_1^{-1}T_2z$.

Or

(b) Prove that the line integral $\int_\gamma pdx + qdy$, defined in Ω, depends only on the end points of γ if and only if there exists a function $U(x, y)$ in Ω with the partial derivatives

$$\frac{\partial U}{\partial x} = p, \frac{\partial U}{\partial y} = q.$$

18. (a) State and prove Cauchy's theorem for a rectangle.

Or

(b) Suppose that $\phi(\xi)$ is continuous on the arc γ. Prove that the function $F_n(z) = \int_\lambda \dfrac{\phi(\xi)\, d\xi}{(\xi - z)^n}$ is analytic in each of the regions determined by γ, and its derivative is $F_n'(z) = n\, F_{n+1}(z)$.

19. (a) State and prove Taylor's theorem.

Or

(b) State and prove the lemma of Schwarz.

20. (a) State and prove the argument principle.

Or

(b) Evaluate $\int_0^\infty \frac{x \sin x}{x^2 + a^2} dx$, a real.